UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 4TH FEBRUARY 1999

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

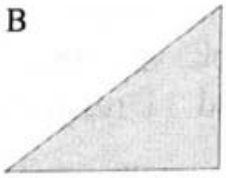
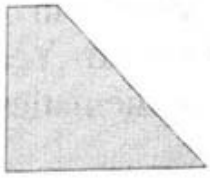
RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 11 or below.
   Candidates in Scotland must be in S4 or below.
   Candidates in Northern Ireland must be in School Year 12 or below.
5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
6. *Do not expect to finish the whole paper in 1 hour.* Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
7. Five marks are awarded for each correct answer to Questions 1-15.
   Six marks are awarded for each correct answer to Questions 16-25.
   Each incorrect answer to Questions 16-20 loses 1 mark.
   Each incorrect answer to Questions 21-25 loses 2 marks.
8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the sheet except to mark your chosen options**. The machine 'sees' all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.
9. The questions on this paper challenge you to **think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.
   The UK IMC is about solving interesting problems, not about lucky guessing.

1. Which of these numbers is biggest?

A 19×99 B 199×9 C 199^9 D $1^9 \times 9^9$ E 1^{999}

2. A sheet of A4 size paper (297mm × 210mm) is folded once and then laid flat on the table. Which of these shapes could not be made?

A  B  C  D  E 

3. A certain company offers "750 hours of free Internet use for new subscribers". On closer inspection it becomes clear that this time must be used during the new subscriber's first month of membership!

What is the maximum number of hours in any one month of the year?

A 168 B 692 C 720 D 744 E 750

4. Ima Divvy used her calculator and multiplied a number by 20 instead of by 2. What could she now do to obtain the correct answer?

A divide by 20 B divide by 40 C multiply by 10
D multiply by 0.5 E multiply by 0.1

5. $30 \div 0.2$ equals

A 1.5 B 6 C 15 D 150 E 600

6. In Britain in 1996 we consumed on average 9.6 kg of bananas per person per year (that is, around 60 bananas each). In some parts of Africa, the consumption of bananas is as high as 250 kg of bananas per person per year. Roughly how many bananas is that?

A 4 or 5 a day B 1 or 2 a day C 4 or 5 a week
D 1 or 2 a week E 4 or 5 a month

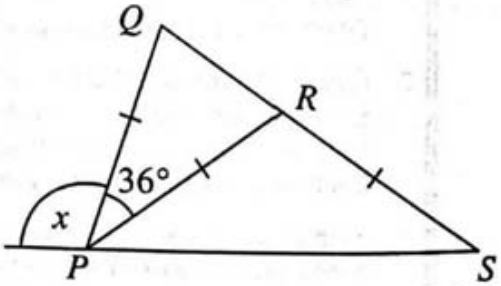
7. Which is smallest?

A $\frac{(2+3)}{(4+6)}$ B $\frac{(2 \div 3)}{(4 \div 6)}$ C $\frac{23}{46}$ D $\frac{(2-3)}{(4-6)}$ E $\frac{(2 \times 3)}{(4 \times 6)}$

8. In the diagram $PQ = PR = RS$.

What is the size of angle x?

A 54° B 72° C 90° D 108° E 144°



9. It is evening and Meg, who is 1m tall, casts a shadow of length 3m. If Meg stands on her brother's shoulders, which are 1.5m above the ground, how long a shadow will she and her brother cast?

A 3m B 4.5m C 5.5m D 6.5m E 7.5m

10. In March 1998 a book called "*The Shadow of the East*" was returned to a library in Sussex. It had been borrowed on January 3rd 1924! The library charges a fine of 60p per week for overdue books. Approximately how big a fine should the person who returned the book have paid?

A £45 B £180 C £230 D £2200 E £16 000

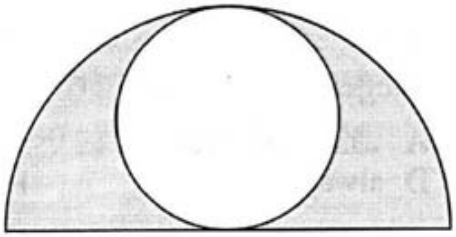
11. "20% off everything", screamed the sale posters. I paid £60. What would I have paid before the sale?

A £60 B £66 C £72 D £75 E £80

12. In a right angled triangle the two shorter sides have lengths 10cm and 5cm. Which of the following approximations is closest to the length of the hypotenuse?

A 11cm B 11.5cm C 12cm D 12.5cm E 13cm

13. The diagram shows a semicircle containing a circle which touches the circumference of the semicircle and goes through its centre. What fraction of the semicircle is shaded?

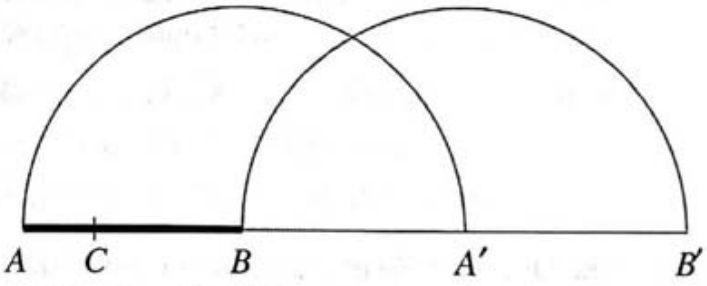


A $\frac{2}{3}$ B $\frac{1}{2}$ C $\frac{1}{\pi}$ D $\frac{2}{\pi}$ E $\frac{3}{\pi}$

14. Which of the following statements is false?

A an octagon has twenty diagonals
B a hexagon has nine diagonals
C a hexagon has four more diagonals than a pentagon
D a pentagon has the same number of diagonals as it has sides
E a quadrilateral has twice as many diagonals as it has sides

15. A pencil AB lying on a table is given a half-turn about the end B (so that A moves to A') and then a half-turn about A' (so that B moves to B'). The point C on the pencil is one third of the way from A to B.



What is the ratio of the total distances moved by A and by C?

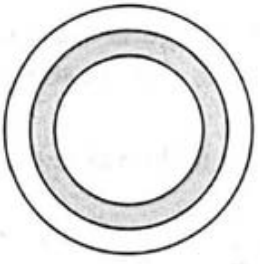
A 3:1 B 3:2 C 1:1 D 2:3 E 1:3

16. On the right are three statements. Exactly which ones are true?

(i) 3^{10} is even (ii) 3^{10} is odd (iii) 3^{10} is square

A (i) only B (ii) only C (iii) only D (i) and (iii) E (ii) and (iii)

17. The three circles in the diagram have the same centre and have radii 3cm, 4cm and 5cm. What percentage of the area of the largest circle is shaded?



A 20% B 25% C 28% D 30% E $33\frac{1}{3}$%

18. Seventy pupils (37 boys and 33 girls) are divided into two groups, with forty pupils in Group I and thirty pupils in Group II. How many more boys are there in Group I than there are girls in Group II?

A 4 B 7 C 8 D 9 E more information needed

19. Four *wiggles* are the same as three *woggles*; two *woggles* are the same as five *waggles*, and six *waggles* are the same as one *wuggle*. Which is smallest?

A 1 *wuggle* B 2 *woggles* C 3 *waggles* D 4 *wiggles* E two have the same value

20. Inspector Remorse estimates that he can solve the average murder in x hours, a bank robbery in half that time, and a car theft in one third of the time he takes to solve a bank robbery. How many hours would he expect to take in solving two murders, six car thefts and four bank robberies?

A $3x$ B $5x$ C $6x$ D $7x$ E $12x$

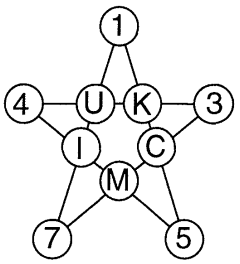
21. When exactly is the value of the product $(1 + \frac{1}{2})(1 + \frac{1}{3})(1 + \frac{1}{4})\ldots(1 + \frac{1}{n})$ equal to an integer?

A when n is odd B when n is even C when n is a multiple of 3
D always E never

22. In the *Soft Boulder Café* each table has 3 legs, each chair has 4 legs and all the customers and the three members of staff have two legs each. There are four chairs at each table. At a certain time, three-quarters of the chairs are occupied by customers and there are 206 legs altogether in the café. How many *chairs* does the café have?

A 20 B 24 C 28 D 32 E 36

23. In the star shown here the sum of the four numbers in any "line" is the same for each of the five "lines". The five missing numbers are 9, 10, 11, 12 and 13. Which number is represented by K?



A 9 B 10 C 11 D 12 E 13

24. The Queen of Hearts has lost her tarts! She is sure that those knaves who have not eaten the tarts will tell her the truth and that the guilty knaves will tell lies. When questioned, the five knaves declare:

K1 "*One of us ate them.*" K2 "*Two of us ate them.*" K3 "*Three of us ate them.*"

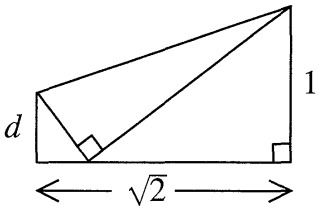
K4: "*Four of us ate them.*" K5: "*Five of us ate them.*"

How many of the knaves were honest?

A 1 B 2 C 3 D 4 E 5

25. A rectangular sheet of paper with sides 1 and $\sqrt{2}$ has been folded once as shown, so that one corner just meets the opposite long edge.

What is the value of the length d?



A $\frac{1}{2}$ B $\sqrt{2} - 1$ C $\frac{7}{16}$ D $\sqrt{3} - \sqrt{2}$ E $\frac{\sqrt{2}}{3}$